THE INSTITUTE OF PHYSICS AND
THE PHYSICAL SOCIETY

Proceedings of a
Conference on

THE SCIENCE OF MATERIALS

17—21 AUGUST 1969

Information Series No. 71

NEW ZEALAND DEPARTMENT OF SCIENTIFIC
AND INDUSTRIAL RESEARCH 1969

THE USE OF THE HERTZIAN FRACTURE TEST FOR MEASURING SURFACE ENERGY VARIATIONS

F. B. LANGITAN and B. R. LAWN, University of New South Wales, Australia

Abstract

In 1882 Hertz demonstrated that the critical normal loading of a hard sphere on to the surface of a brittle solid gives rise to a cone-shaped fracture in the solid. Subsequently Auerbach, in 1891, showed that the critical load P_c varies with the radius r of the indenting sphere according to the simple relation P_c/r = constant (Auerbach's law). Recent theoretical studies of the mechanism of growth of these so-called Hertzian fractures have shown that the proportionality constant in Auerbach's law is directly proportional to the specific fracture surface energy γ of the specimen solid. This is a useful result because it means that information concerning the γ term, an important parameter which determines the inherent strength of ideally brittle solids, may be tabulated simply by recording the critical load required to produce a cone fracture. Moreover, the Hertzian fracture test has many unique advantages over other fracture tests currently employed for measuring the γ term. In this report these advantages will be outlined, and results will be presented showing the manner in which the surface energy of glass varies under a number of test conditions. The strength-controlling mechanisms responsible for such variations will be interpreted in the light of current thought on fracture processes.

INTRODUCTION

In 1882 Hertz published a classic paper describing the nature of deformation suffered by two bodies in mutual contact. In particular, for a hard indenting sphere critically loaded to exceed the elastic limit of a flat surfaced elastic-brittle solid (glass), Hertz observed a cone-shaped crack (the so-called Hertzian cone fracture) to propagate into the specimen solid from the mutual contact circle (Fig. 1). Subsequently, in 1891 Auerbach investigated the dependence of the critical normal load P_c on the radius r of the indenting sphere, and found the linear relation $P_c \propto r$. This empirical relation, now known as Auerbach's law, has received extensive attention because of its deep implications concerning the validity of certain widely accepted fracture criteria commonly used to predict fracture processes in brittle solids (Langitan and Lawn 1969).

It is only recently that the mechanics of Hertzian fracture have been satisfactorily accounted for (Langitan and Lawn 1969; Frank and Lawn 1967; Lawn 1968). To treat the problem quantitatively one has to apply the fundamental Griffith energy balance criterion (Griffith 1920) for fracture (firmly based on the principle of conservation of energy) in a stepwise manner to the growth of the cone crack as it propagates through the strongly inhomogeneous stress field below the ball indenter. In this way it is predicted that the fracture process initiates from a flaw located close to the contact circle between sphere and specimen, and forms a

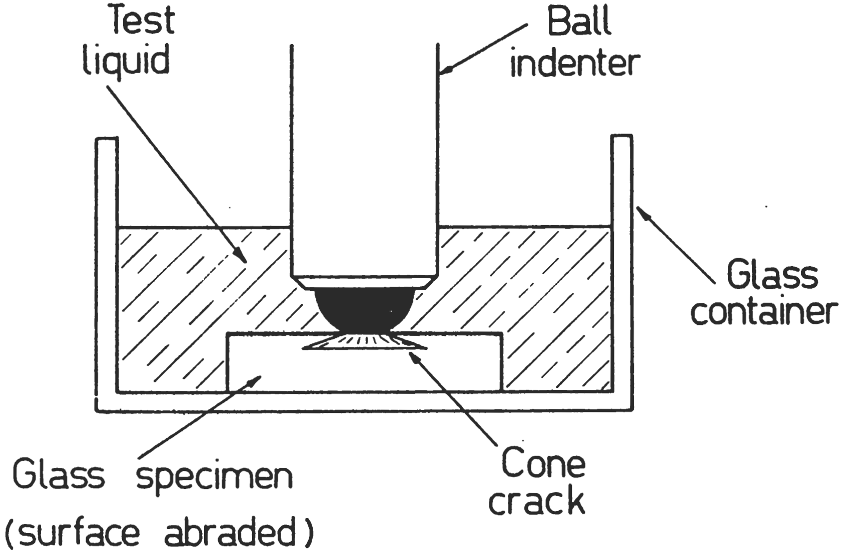

FIG. 1—Experimental arrangement for producing cone fractures in an immersed glass specimen.

stably propagating surface ring crack (not yet visible) as the indenter load is steadily increased. At a critical load the surface ring suddenly becomes unstable and propagates (without further increase in the load) into the specimen to form the completed cone crack. It is this event which is visually observed in an experiment, and the calculated condition for it to occur is

$$P_c = K(E)\,\gamma\, r \tag{1}$$

where P_c and r are as above, K(E) is a constant containing elastic constants, and γ is the *reversible specific fracture surface energy* of the specimen solid. The cone crack thus formed can be made to extend (stably) further into the specimen by steadily increasing the load beyond P_c. Equation (1) provides a rigorous theoretical confirmation of Auerbach's law; it has been shown to be valid for highly brittle single crystals (Lawn 1968) as well as for glasses (Frank and Lawn 1967).

Of particular interest to this paper is the appearance of γ in equation (1). It is this fracture surface energy term which controls the *inherent* strength of brittle solids. The Hertzian fracture test thereby affords, via equation (1), a simple and convenient means for investigating the strength properties of glasses, with many unique advantages over alternative testing methods (e.g. cross-bending test, cantilever-cleavage test, etc.). In our laboratories a full scale study programme on the many factors affecting glass strength has been proposed using the Hertzian method, and some preliminary results will be presented here. These results will be discussed briefly in terms of current theories of fracture processes in glassy solids.

The Hertzian Test as a Means of Measuring Surface Energies

The standard methods available for tabulating information about γ (of which the cantilever-cleavage technique is probably now the most widely used) cannot yet be considered to give always reliable results, especially as far as the determination of absolute values is concerned. Unfortunately the constant K(E) in equation (1) is itself subject to considerable uncertainty in the theoretical treatment of cone crack growth; however, though this precludes an accurate estimate of the absolute value of γ for a given material, it does not prevent an accurate measure of *relative* values of γ. Thus, if we perform Hertzian fracture tests with a ball of selected size on a given specimen subjected to a variety of test conditions, we should be able to follow any variations in the γ term with the same accuracy with which we can measure P_c, provided the elastic constants of indenter and specimen materials remain invariant between experiments (usually a reasonable assumption).

In this regard the Hertzian fracture test has several advantageous features over alternative techniques:

(1) Experiments are simple to perform. A commercially produced hardened steel ball (ball-bearing type) is quite suitable for indentation tests on glass. Each test takes approximately 10 sec on a standard testing machine, and the appearance of the cone crack can be observed visually in a transparent material.

(2) Well over 100 tests can be made on one specimen surface measuring 1 in. × 1 in. Thus the inherent strength of a material can be investigated on the one specimen under several test conditions. Other techniques require separate experiments to be carried out on separate specimens, the very nature of the fracture in these cases usually leading to the destruction of the original specimen. This is an important point since it is difficult to reproduce strength values from specimen to specimen, especially when the samples are taken from different glass batches.

(3) It is noted that the critical load in equation (1) does not involve the size of the original flaw from which the cone crack initiates, nor does it involve a knowledge of the length of the cone crack at any stage along its path: the value of P_c simply refers to the load at which the cone crack suddenly appears. In all other conventional tests the fracture strength involves a crack length as well as γ.

(4) Since the cone crack begins as a stably propagating surface ring the γ term in equation (1) represents the *reversible* fracture surface energy. While the cantilever-cleavage method also uses a stably propagating crack (if the cleavage crack-opening is displacement-controlled), the same is not true for the cross-bending test in which the fractures initiate spontaneously and catastrophically from a flaw.

(5) With careful specimen surface preparation the value of P_c as measured for a given experimental condition can be reproduced to

within an unusually high degree of accuracy. Thus, by carefully abrading the specimen surface, thereby producing a uniform layer of surface flaws to serve as nucleation sites for the cone fractures, P_c may be reproduced to better than 5% (Langitan and Lawn 1969).

Experimental Procedures for Tests on Plate Glass

Hertzian fracture experiments were carried out on plate glass slabs (Pilkington, Australia), 2 in. × 2 in. × ½ in., the specimen surfaces being first abraded with a slurry of No. 400 SiC abrasive powder. A ½ in. diameter steel ball was used as indenter, this being mounted in the underside of the crosshead of an Instron testing machine. The specimens were seated beneath the indenter on a compression load cell of the testing machine, and the load corresponding to the incidence of fracture in the glass slab was measured on a pen-recorder. For each test a series of 10 fractures was made, from which a mean value, and the standard deviation about this mean, could be computed for P_c.

It is well known that the strength of glass is strongly dependent on the surrounding environment. In particular the presence of water vapour is known to have a considerable effect on strength values. Our first experiments have thereby been directed toward investigating the effects of the presence of an environmental liquid on the γ term. For this test the specimens were simply immersed in a container of liquid, as shown in Fig. 1. With the load on the indenter concentrated just below the indenter itself the cone crack (readily visible through the side walls of the container) forms well before the glass container (which supports the indenter load over the entire specimen base) is itself in danger of cracking.

Effect of Specimen History on Strength in Air and Water

Several tests have been made on specimens first open to room atmosphere (controlled at 20°c, 60% humidity), and then immersed in distilled water. The indented specimens were subsequently annealed for 24 hours at each of several successive temperatures, the air/water fracture test being repeated after each anneal. As seen in Fig. 2 the effect of immersing the specimens in water is to reduce the air value of the critical load by about 15%, whereas the effect of the annealing process on the data is insignificant.

The result in Fig. 2 indicates that the γ term is strongly dependent on the presence of water, but that it is independent of the manner in which the specimen surface is treated ("aged") between abrasion and indentation. These results are in contrast to the strong dependence of fracture strength on ageing shown by similarly abraded specimens subjected to *cross-bending* (Marsh 1964). The difference in behaviour exhibited by the two tests may be attributed to the fact that the strength in the bending test is a function of the size and severity of the flaw responsible for the

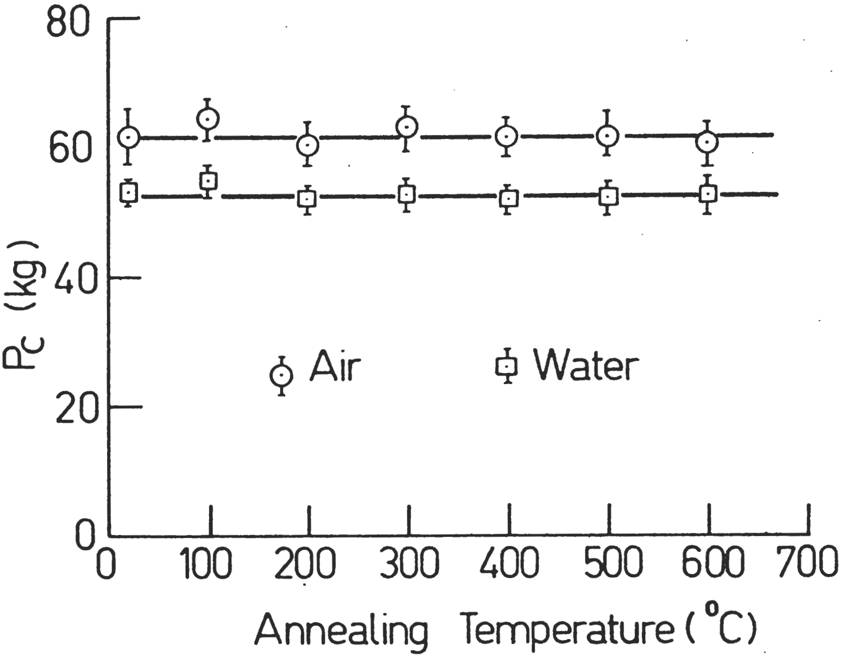

FIG. 2—Critical Hertzian fracture load as function of annealing temperature for specimens tested in air and in water.

ultimate fracture, as well as on γ. Thus, taking the results of the two tests together, it would appear that the dependence of the bending strength on specimen history is related to the effects of ageing processes on flaw size (e.g. on flaw "healing"). It is pointed out here that from the bending test results alone nothing can be inferred as to the effect of specimen history on the γ term.

We may conclude from the data in Fig. 2 that the fracture surface energy is strongly dependent on the environment at the *time of testing* but very little on the environment *before testing*. Thus the effect of water on the strength of glass would appear to be largely reversible. To investigate this effect a stage further, a specimen was tested in air, then in distilled water, and finally, after drying the specimen between sheets of blotting paper, in air again, all at room temperature. While the test in water showed the usual depreciation in P_c, the second test in air gave a result agreeing (within experimental error) with the original test value in air.

EFFECT OF ACIDIC-BASIC SOLUTIONS

It has been suggested that the fracture properties of glass are affected by chemical attack at the fracture interface by the surrounding medium. In this respect Mould (1960) demonstrated that an acidic solution would be more prone to enhance fracture than would be a basic one. If such an effect were to be attributable solely to the effects of such attack on the geometry of the flaws, rather than to a genuine variation in γ, we should expect the Hertzian fracture load to be independent of the test conditions (as we found for the annealing treatment above). The results of a study of the effect of the pH of a solution on P_c, illustrated in Fig. 3, do not show such constancy. On the other hand the trend shown in Fig. 3, with pronounced weakening and strengthening effects at opposite ends of the pH scale, is in accord with that found by Mould. The interpretation of this curve would appear to be closely connected with the chemical effects of the test solution on the strength of glass in the vicinity of the crack tip; that is with a direct influence of the solution on the fracture surface energy.

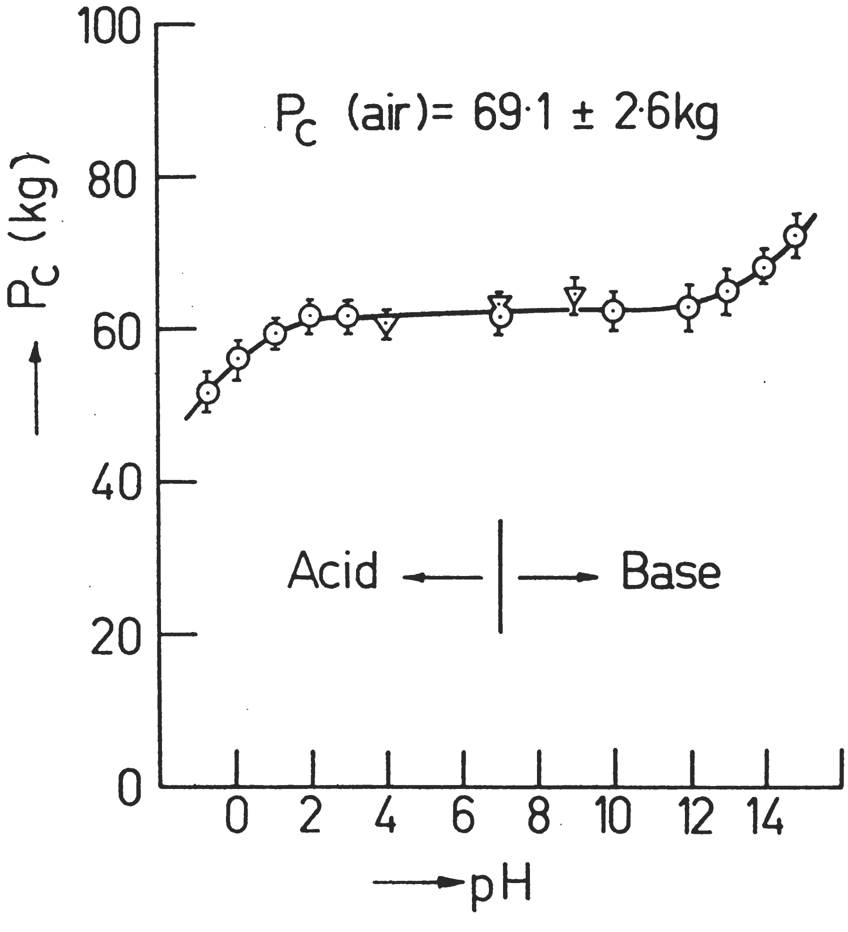

FIG. 3—Critical Hertzian fracture load as function of solution pH. Acid solutions (HCl) and basic solutions (NaOH) denoted by circles, buffer solutions by triangles.

Effect of Other Liquids

A good deal of information concerning the effects of different liquid surroundings on the strength of glass has been reported in the literature. Comprehensive studies on this topic are rare, however, since bending tests (which provide most of the available data) require a vast number of specimens in order to attain sufficient statistical accuracy, while those methods which involve the direct measurement of the effect of environment on crack length (Culf 1957; Linger and Holloway 1968) have other specimen limitations. Consequently, the literature contains a large number of data which are, by virtue of uncontrollable differences in experimental conditions, often difficult to correlate satisfactorily.

Thus the effect of liquid (and gas) environment on glass strength is a topic ideally suited for study by the present method, in which several series of experiments can be performed on the one specimen surface. Some results in this direction are listed in the table. These results were obtained by immersing the test specimen in liquids (as in Fig. 2) without any attempt being made to remove traces of water from the liquid. Since even small traces of water are suspected of largely influencing the strength values, these results must be considered to be preliminary. The experiments are to be repeated in due course with carefully "dried" liquids; this awaits more sophisticated techniques than are available to us at present. Nevertheless the results in the table show significant effects. It is evident that glass is stronger when tested in the organic liquids listed than when tested in air, indicating higher values of γ.

Table—Critical Load to Produce Cone Cracks in Abraded Glass Specimens with $\frac{1}{2}$ in. Diameter Ball Using Different Liquid Environments. Tests in Air at 20°c, 60% Humidity

Liquid Environment	P_c (kg)			Ratio
	Air (before)	Liquid	Air (after)	$\frac{\gamma \text{ liq}}{\gamma \text{ air}}$
Water	69.8 ± 2.2	62.6 ± 2.3	69.8 ± 1.8	0.90
Benzene	66.7 ± 2.7	87.6 ± 4.5	67.0 ± 2.7	1.31
Ether	68.5 ± 2.9	83.6 ± 3.4	71.3 ± 2.6	1.20
Methanol	69.0 ± 2.9	83.7 ± 2.3	67.4 ± 2.2	1.23
Acetone	63.8 ± 2.6	74.9 ± 2.8	64.6 ± 2.6	1.17

Proposed Further Studies

At present a chamber is being constructed which will house the test specimens and permit the Hertzian test to be carried out in vacuum (10^{-6}–10^{-7} torr), or in carefully controlled liquid and gas environment. With this experimental modification some light should be thrown on the general fatigue behaviour of glass. For instance, it is known that glass exhibits "delayed" fracture in certain environments, i.e. fracture occurs when the specimen is sustained for long periods at loads considerably less than those needed to cause "instantaneous" fracture. This effect is not observed in vacuum. The effect of temperature on the γ term is also to be studied by incorporating a hot-stage into the apparatus. Further, it is intended to study the possible effects of radiation processes, both particle and photon, on γ.

Finally, when the techniques for studying glass have been perfected, the extension of the study to (more expensive) single crystals is proposed.

Discussion of Results in Terms of Fracture Theory

Several theories have been proposed to account for the fatigue properties of glass. Since the strength of a brittle solid *in general* depends on two parameters, the fracture surface energy γ (an inherent property of the solid) and the size and severity of pre-existing cracks (a function of the previous mechanical history of the solid), these theories may be conveniently subdivided into two categories:

(1) Those relating strength to the effect of environment on the fracture surface energy (Orowan 1944). The availability of the environmental species to the root of the crack tip determines the *rate* at which this effect occurs, thus explaining the time-dependent nature of fatigue.

(2) Those relating strength to the effect of environment on the geometry of the crack tip configuration. The reacting species may diffuse into the glass near the crack tip (Elliot 1958), thereby causing a structural change in the glass there. Alternatively, the reaction may be a stress-corrosion process at the crack tip, in which the geometry of the crack tip is altered (Hillig 1962).

The variants on the above are numerous, and it is indeed often difficult to fit a particular theory into any one specific category. The possibility of plastic flow at the crack tip is a case in point (Marsh 1964; Linger and Holloway 1968): in one sense plastic flow would effectively increase the fracture surface energy, while in another it would serve to blunt the tip of the crack; both lead to an increased fracture strength. This aside, it is seen that the Hertzian test is, in principle, the only existing fracture test with the potential capability of distinguishing between the two categories in a direct manner. For instance, in the annealing experiment (Fig. 2) it is evident that heating the specimens has not affected the fracture surface energy of the glass. In the bending test, however, the glass strength depends strongly on such treatment, in which case no direct conclusion can be drawn as to the correct theoretical category from these results alone. Again, the presence of different liquids at the crack interface strongly affects the Hertzian fracture load, thereby indicating a corresponding (reversible) effect on γ, while in the bending test (although the results show similar trends) the interpretation is less positive. Clearly, considerably more information needs to be collected before the fatigue properties of glass can be better understood, but the Hertzian test nevertheless provides a particularly promising means for collecting much of this information.

References

AUERBACH, F. 1891: *Ann. phys. Chem. 43:* 61.

CULF, C. J. 1957: *J. Soc. Glass Technol. 41:* 157.

ELLIOT, H. A. 1958: *J. appl. Phys. 29:* 224.

FRANK, F. C.; LAWN, B. R. 1967: *Proc. R. Soc., Lond. A229:* 291.

GRIFFITH, A. A. 1920: *Phil. Trans. R. Soc. A221:* 163.

HERTZ, H. 1882: Verhandlungen des Vereins zur Beforderung des Gewerbe Fleisses *61:* 449. (Reprinted in English, in Hertz's Miscellaneous Papers, Ch. 6, London, Macmillan.)

HILLIG, W. B. 1962: "Sources of Weakness and the Ultimate Strength of Brittle Amorphous Solids". *In:* Mackenzie, J. D., "Modern Aspects of the Vitreous State", p. 152. Vol. 2, Butterworths, Washington.

LANGITAN, F. B.; LAWN, B. R. 1969: *J. appl. Phys.:* (In press).

LAWN, B. R. 1968: *J. appl. Phys. 39:* 4828.

LINGER, K. R.; HOLLOWAY, D. G. 1968: *Phil. Mag. 18:* 1269.

MARSH, D. M. 1964: *Proc. R. Soc., Lond. A 279:* 420; *A282:* 33.

MOULD, R. E. 1960: *J. Am. Ceram. Soc. 43:* 160.

MOULD, R. E. 1961: *J. Am. Ceram. Soc. 44:* 481.

OROWAN, E. 1944: *Nature 154:* 341.