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Effect of Residual Contact Stresses on Mirror/Flaw-Size Relations

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THE characterization of flaws is an important aspect of strength analysis in brittle solids. Generally, flaw origins on fracture faces are surrounded by a well-defined morphological pattern, consisting of distinctive "mirror," "mist," and "hackle" regions.[1] From several studies of this phenomenon, initially in glasses[2–5] but more recently in polycrystalline ceramics,[6,7] each of the mirror dimensions, r_M (mirror/mist), r_H (mist/hackle), and r_B (branching), is found to relate empirically to the applied stress at failure (σ_f)

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according to

$$\sigma_a r^{1/2} = A = \text{constant} \quad (1)$$

The form of this relation implies that each feature of the morphology is determined by a specific stress intensity factor.[4–6,8] Since the flaw size c at failure bears a similar relation to the applied stress, it appears reasonable to assert that the ratio r/c should be constant, i.e. the fracture mirror should provide a scaled-up "image" of the flaw. This result, confirmed in several investigations,[5,6,9–11] is particularly useful in cases where the flaw origin is too small for detailed microscopic examination; knowledge of the mirror constants for the fractured material should then allow the flaw size, as well as the failure stress, to be evaluated directly from measurements of r.

A tacit assumption in establishing these relations for the mirror pattern is that the material be free of extraneous stresses across the prospective crack plane. This assumption does not always hold in practice. For instance, significant residual compressive stresses may be generated in glasses and ceramics cooled rapidly from high temperatures. However, provided the residual compression σ_R is reasonably uniform over the area of the mirror, Eq. (1) remains valid if σ_a is replaced by an "effective failure stress" $\sigma_a' = \sigma_a - \sigma_R$.[1,6,7] Indeed, fracture-mirror measurements have been used to determine the actual levels of residual stress in surface-strengthened glass rods.[12] The preservation of homogeneity in the net field on the crack means that the mirror/flaw-size ratio r/c is unchanged, suggesting a certain universality in the fracture morphology.

On the other hand, the typical flaw may experience other components of residual stress, especially those associated with the processes actually responsible for the generation of the flaw itself,[13] which may be far from homogeneous. Such is the case with the deformation-induced surface microcracks produced in sharp-point contact.[14] The inhomogeneity of the residual field can then significantly alter the mechanics of failure.[15] Since the strength of a great many brittle solids is controlled by contact flaws, it is pertinent to investigate the influence of this additional factor on the mirror/flaw-size relations.

A theoretical description of the prospective flaw response may be obtained from indentation fracture mechanics. The flaw is taken to possess the penny-like geometry of the median/radial crack system,[16] with an initial radius c_0 determined by the contact load P. It is convenient to consider the cases of zero and nonzero residual-stress terms separately:

(1) Zero residual contact stresses: If the indentation flaw were to be free of residual stresses, the stress intensity factor for uniform tensile loading would have the standard form

$$K = Y\sigma_a c^{1/2} \quad (2)$$

at all $c \geq c_0$, where Y is a dimensionless geometrical factor. The requirement for failure is that $K = K_c$, at which point the flaw expands spontaneously. This situation may be realized under equilibrium or nonequilibrium fracture conditions; in the latter case some subcritical crack growth precedes the instability. Regardless of which conditions prevail, the flaw size at failure is given by

$$c^* = (K_c/Y\sigma_a^*)^2 \quad (3)$$

where the asterisk notation is used to denote critical values of the test variables. The unstable crack accelerates at constant stress σ_a^* until, in accordance with Eq. (1), the stress intensity reaches a level $K = K_r$ at which the mist- and hackle-producing processes begin to operate. This defines the characteristic mirror radii at $c = r$ (Fig. 1),

$$r = (K_r/Y\sigma_a^*)^2 \quad (4)$$

Assuming that Y remains invariant, Eqs. (3) and (4) combine to give a constant mirror/flaw-size ratio,

$$r/c^* = (K_r/K_c)^2 \quad (5)$$

(2) Nonzero residual contact stresses: With residual contact stresses present at the flaw origin it becomes necessary to include an additional tensile term in the stress intensity factor[14,15]:

$$K = Y\sigma_a c^{1/2} + \chi_r P/c^{3/2} \quad (6)$$

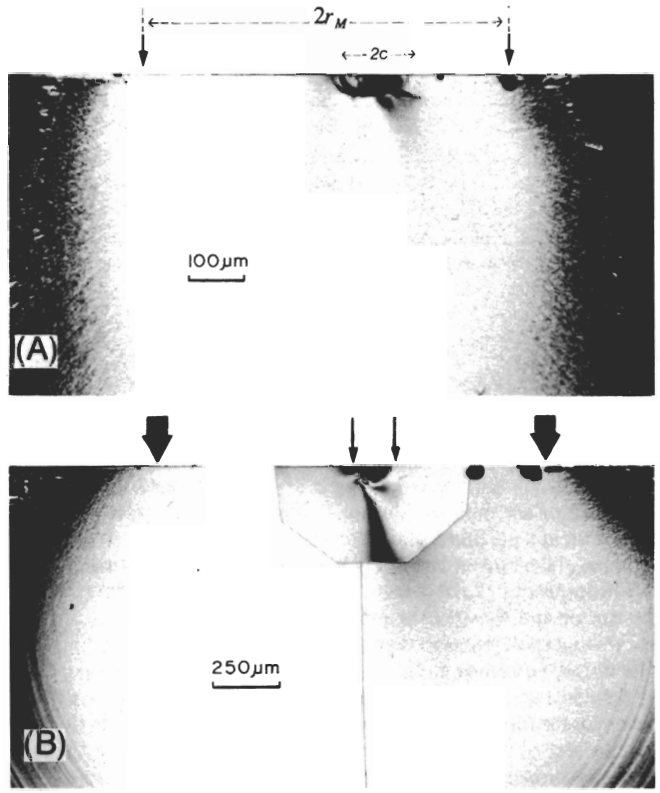

Fig. 1. Optical fractographs of flaw/mirror zones on soda-lime glass surfaces. (*A*) Annealed specimen, oil environment, $P = 5$ N, $\sigma_a^* = 99$ MPa; (*B*) mosaic of as-indented specimen, oil environment, $P = 5$ N, $\sigma_a^* = 66$ MPa. Thin arrows indicate critical flaw boundary, broad arrows represent mirror/mist boundary. Note scale of crack morphology relative to specimen thickness (3 mm).

where χ_r is a dimensionless constant which defines the magnitude of the residual field for a given indenter/material system. The function $K(c)$ in Eq. (6), in contrast to its monotonically increasing counterpart in Eq. (2), has a stationary value; whereas at large c the applied-stress term controls the fracture, as before, at small c it is the residual-stress term which dominates. Under equilibrium fracture conditions the flaw will accordingly undergo a precursor stage of stable growth as the tensile loading is applied; failure then occurs when the crack reaches a critical size, at which point the applied stress is intense enough to cause spontaneous propagation.[14,15] This critical configuration is obtained by inserting $K = K_c$ into Eq. (6) and evaluating the instability condition $d\sigma_a/dc = 0$:

$$c^* = (4\chi_r P/K_c)^{2/3} = (3K_c/4Y\sigma_a^*)^2 \quad (7)$$

Once the crack front is distant from the flaw origin, $r >> c^* > c_0$, the residual term in Eq. (6) becomes negligibly small, in which case Eq. (4) remains a good approximation of the mirror dimensions. Combination of Eqs. (4) and (7) then gives the mirror/flaw-size ratio

$$r/c^* = (4K_r/3K_c)^2 \quad (8)$$

which differs from the result in Eq. (5) by a factor 1.78. Under nonequilibrium conditions, on the other hand, the flaw may extend subcritically to a radius in excess of that representing the stationary configuration in Eq. (7),[15] whence the residual term exerts a diminished influence on the critical dimension c^* at failure; a ratio between those in Eqs. (5) and (8) would then be expected.

To investigate the preceding predictions, a Vickers pyramid was used to produce well-defined, median/radial indentation flaws in soda-lime glass disks (50 mm in diam. by 3 mm thick). The disks were then broken in concentric ring-on-ring loading, with the inden-

Table I. Mirror Constant for Vickers Flaws in Soda-Lime Glass

Boundary	A (MPa·m$^{1/2}$) for specimen/environment indicated*		
	Annealed/oil	As-indented/oil	As-indented/water
Mirror/mist	1.82± 0.04	1.94± 0.02	1.92± 0.04
Mist/hackle	2.03± 0.04	2.13± 0.04	2.08± 0.04
Branching	2.28± 0.06	2.42± 0.08	2.30± 0.05

*Values given as mean and standard deviation.

Table II. Mirror/Flaw-Size Ratio for Vickers Flaws in Soda-Lime Glass

Boundary	r/c^* for specimen/environment indicated				
	Annealed/oil	As-indented/oil		As-indented/water	
	(Abs.)	(Abs.)	(Norm.)	(Abs.)	(Norm.)
Mirror/mist	5.0±0.3	9.0±0.8	1.79±0.17	6.6± 0.4	1.32± 0.08
Mist/hackle	6.2± 0.5	10.9±1.0	1.74± 0.10	7.7± 0.5	1.24± 0.08
Branching	7.8± 0.6	13.9±1.4	1.77± 0.12	9.2± 0.8	1.18± 0.10

*Values given as mean and standard deviation; abs. = absolute ratio, norm. = ratio normalized to annealed/oil data.

tation site in the center of the tensile face. Some of the disks were subjected to an intermediate annealing process between indentation and failure-testing to remove the residual contact stresses ($\chi_r = 0$); the remainder were tested in their as-indented state ($\chi_r > 0$).[14,15] Equilibrium and nonequilibrium failure conditions were obtained by testing in paraffin oil and water environments, respectively. The sections of the broken test pieces were then examined optically. The examples in Fig. 1 show the distinctive markings which delineate the mirror and flaw dimensions at failure.

Fractographic measurements are summarized in Tables I and II. The mirror constants A, Eq. (1), are indeed relatively independent of flaw history, as predicted by theory. (Small systematic differences for the different flaw treatments may be attributable to the fact that the size of the mirror in some cases was an appreciable fraction of the specimen thickness, Fig. 1; note from Eq. (4) that $A = K_r/Y$, so thickness effects may enter via the geometrical term Y.) The values for the mirror/mist and mist/hackle boundaries listed in Table I may be compared with $A_M = 1.9$ MPa m$^{1/2}$ and $A_H = 2.2$ MPa m$^{1/2}$ from earlier determinations on soda-lime glass.[5,10,11]

In contrast, the mirror/flaw-size ratios, r/c^*, are highly sensitive to flaw history. Taking ratios appropriate to residual-stress-free, equilibrium flaw growth (i.e. annealed specimens, oil test environment) as base-line quantities for normalizing the data for flaws subject to nonzero residual stress (i.e. as-indented specimens), factors close to those theoretically predicted, viz. 1.78 for equilibrium growth (oil environment) and within the range 1.00 to 1.78 for nonequilibrium growth (water environment), are obtained. When compared with previous determinations for semicircular grinding flaws in soda-lime glass test pieces taken to rapid failure[17] (i.e. negligible subcritical growth), $r_M/c^* = 10.0$ and $r_H/c^* = 11.8$, the absolute values in Table II appear to be low. However, if a correction factor of ≈ 1.2 is introduced to accommodate the ellipticity of the flaws in Fig. 1 (by replacing the radius c with the geometrical mean of the semimajor and semiminor axes),[17,18] the values for the as-indented, oil-tested specimens approach the earlier results. This finding suggests that grinding flaws are also subject to residual-stress effects and, in this sense, may be regarded as a special case of deformation-induced indentation cracking.[19]

It is therefore apparent that, whereas residual contact stresses are not a major factor in the mechanics of mirror formation, they can exert a strong influence on the instability conditions which take the flaw to failure. The use of mirror constants in the evaluation of initial flaw sizes accordingly demands that due attention be given to specimen history.

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[1] J. J. Mecholsky, S. W. Freiman, and R. W. Rice; pp. 363–79 in Fractography in Failure Analysis. Edited by B. M. Strauss and W. H. Cullen. *Am. Soc. Test. Mater., Spec. Tech. Publ.*, No. **645**, 1978.

[2] W. C. Levengood, "Effects of Original Flaw Characteristics on Glass Strength," *J. Appl. Phys.*, **29** [5] 820–26 (1958).

[3] E. B. Shand, "Breaking Stress of Glass Determined from Dimensions of Fracture Mirrors," *J. Am. Ceram. Soc.*, **42** [10] 474–77 (1959).

[4] J. W. Johnson and D. G. Holloway, "Shape and Size of Fracture Zones on Glass Fracture Surfaces," *Philos. Mag.*, **14** [130] 731–43 (1966).

[5] J. J. Mecholsky, R. W. Rice, and S. W. Freiman, "Prediction of Fracture Energy and Flaw Size in Glasses from Measurements of Mirror Size," *J. Am. Ceram. Soc.*, **57** [10] 440–43 (1974).

[6] J. J. Mecholsky, S. W. Freiman, and R. W. Rice, "Fracture Surface Analysis of Ceramics," *J. Mater. Sci.*, **11** [7] 1310–19 (1976).

[7] (*a*) H. P. Kirchner and R. M. Gruver, "Fracture Mirrors in Alumina Ceramics," *Philos. Mag.*, **27** [6] 1433–46 (1973).

(*b*) H. P. Kirchner and R. M. Gruver; pp. 309–21 in Fracture Mechanics of Ceramics, Vol. 1. Edited by R. C. Bradt, D. P. H. Hasselman, and F. F. Lange. Plenum, New York, 1974.

(*c*) H. P. Kirchner, R. M. Gruver, and W. A. Sotter, "Fracture Stress-Mirror Size Relations for Polycrystalline Ceramics," *Philos. Mag.*, **33** [5] 775–80 (1976).

[8] H. P. Kirchner and J. W. Kirchner, "Fracture Mechanics of Fracture Mirrors," *J. Am. Ceram. Soc.*, **62** [3–4] 198–202 (1979).

[9] D. A. Krohn and D. P. H. Hasselman, "Relation of Flaw Size to Mirror in the Fracture of Glass," *ibid.*, **54** [8] 411 (1971).

[10] S. W. Freiman, A. C. Gonzalez, and J. J. Mecholsky, "Mixed-Mode Fracture in Soda-Lime Glass," *ibid.*, **62** [3–4] 206–208 (1979).

[11] J. J. Mecholsky, A. C. Gonzalez, and S. W. Freiman, "Fractographic Analysis of Delayed Failure in Soda-Lime Glass," *ibid.*, [11–12] 577–80.

[12] J. J. Mecholsky and M. G. Drexhage, "Comparison of Optical and Fractographic Measurement of Residual Stress in Compressively Clad Glass Rods"; this issue, pp. 347–49.

[13] B. R. Lawn and T. R. Wilshaw, Fracture of Brittle Solids; Ch. 2. Cambridge University Press, London, 1975.

[14] B. R. Lawn and D. B. Marshall, "Residual Stress Effects in Failure from Flaws," *J. Am. Ceram. Soc.*, **62** [1–2] 106–108 (1979).

[15] (*a*) D. B. Marshall and B. R. Lawn, "Residual-Stress Effects in Sharp-Contact Cracking: I," *J. Mater. Sci.*, **14** [8] 2001–12 (1979).

(*b*) D. B. Marshall, B. R. Lawn, and P. Chantikul, "Residual Stress Effects in Sharp Contact Cracking: II," *ibid.*, [9] 2225–35.

[16] B. R. Lawn, A. G. Evans, and D. B. Marshall, "Elastic/Plastic Indentation Damage in Ceramics: I"; to be published in the *Journal of the American Ceramic Society*.

[17] J. J. Mecholsky, S. W. Freiman, and R. W. Rice, "Effect of Grinding on Flaw Geometry and Fracture of Glass," *J. Am. Ceram. Soc.*, **60** [3–4] 114–17 (1977).

[18] G. K. Bansal, "Effect of Flaw Shape on Strength of Ceramics," *ibid.*, **59** [1–2] 87–88 (1976).

[19] M. V. Swain; pp. 257–72 in Fracture Mechanics of Ceramics, Vol. 3. Edited by R. C. Bradt, D. P. H. Hasselman, and F. F. Lange. Plenum, New York, 1978.