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Fatigue analysis of brittle materials using indentation flaws

Part 1 *General theory*

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A two-part study has been made of the fatigue characteristics of brittle solids using controlled indentation flaws. In this part a general theory is developed, with explicit consideration being given to the role played by residual contact stresses in the fracture mechanics to failure. The distinctive feature of the formulation is a stress intensity factor for well-defined indentation cracks, suitably modified to incorporate the residual component. Taken in conjunction with a standard power-law crack velocity function, this leads to a differential equation for the dynamic fatigue response of a given material/environment system. Reduced variables are then introduced to facilitate generation of "universal" fatigue curves, determined uniquely by the crack velocity exponent, *n*. A scheme for using these curves to evaluate basic fracture parameters from strength data is outlined. In this way the foundation is laid for lifetime predictions of prospective brittle components, as well as for reconstruction of the crack velocity function. One of the major advantages of the analysis is the manner in which the residual stress parameters are accommodated in the normalized fracture mechanics equations: whereas it is understood that *all* strength data are to be taken from test pieces in their as-indented state, so making it unnecessary to have to resort to inconvenient stress-removal procedures between the contact and failure stages of testing, *a priori* knowledge of the residual stress level is not required. The method is proposed as an economical route to materials evaluation and offers physical insight into the behaviour of natural flaws.

1. Introduction

The tendency for brittle glasses and ceramics to exhibit limited lifetimes under conditions of sustained loading is a direct manifestation of chemically-assisted flaw growth [1]. A proper study of so-called "fatigue" behaviour is accordingly an essential element of structural design with brittle components. Central to most modern-day analyses of fatigue is the fracture mechanics concept of a well-behaved "Griffith flaw" which extends from its initial size to a failure configuration in accordance with some specifiable crack velocity function. Unfortunately, *a priori* knowledge of the flaw parameters needed for long-term lifetime prediction is generally unavailable, so it becomes necessary to characterize the flaw population in terms of data from control strength tests. This line of attack suffers from two major drawbacks: first, because of a common tendency to a large scatter in flaw severity from specimen to specimen with as-received surfaces, an inordinately large number of tests has to be carried out in order to obtain statistically meaningful data for materials evaluation; second, because identification of the specific flaw ultimately responsible for failure is usually possible by test-piece examination only after the event, the

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crack evolution can not be followed directly. The trend in engineering design has been for data manipulation to supplant physical insight as the underlying basis of fatigue analysis.

One way of avoiding the problems of variability in strength testing is to introduce dominant flaws by controlled indentation [2–4]. Recent studies in these laboratories along such lines have been made on the dynamic [5] (constant stress rate) and static [6] (constant stress) fatigue properties of soda-lime glass in the presence of water. In these studies the high degree of data reproducibility asssociated with the "indentation/strength" approach proved to be particularly useful for the determination of accurate kinetic constants in the crack velocity equation. More importantly, from detailed observations of the crack response throughout its history prior to test-piece failure [4, 5], it was unequivocally demonstrated that flaws could behave in a manner quite unlike that of the classical Griffith sense, with important implications in lifetime prediction [6]. Specifically, cracks formed in an elastic–plastic contact field using a sharp indenter are subject to a residual driving force [3, 7] and this residual force has a profound influence on the fracture mechanics to failure. Such effects would be felt in any naturally occurring flaw whose nucleation forces persisted in whole or in part in the material [8].

In this investigation, presented in two parts, we illustrate how the indentation/strength procedure can be used to determine the fatigue characteristics of brittle materials. Part 1 outlines the general theory, and Part 2 presents a case study on a "typical ceramic". Whereas in our previous fatigue studies on soda-lime glass [5, 6] emphasis was placed on establishing the validity of the fracture mechanics formulation, especially with regard to the residual stress component, in this work we concern ourselves more with the practical issue of materials evaluation. In the interest of experimental simplicity, the fatigue data are assumed to be taken exclusively from test-pieces in the "as-indented" state: the analytical complexity that attends the consequent need to deal with the residual stresses is considered more than offset by not having to remove these stresses by physical means (e.g. by annealing or surface grinding [9, 10], processes which could significantly alter the very nature of the flaws, or even of the material itself). Moreover, this additional complexity can be minimized by judicious choice of reduced variables, so that the need to specify an appropriate residual-stress parameter is avoided. Adoption of a Vickers hardness indenter for introducing the starting flaws, and of constant stress rates in the breaking routine, allows for convenient standardization of the procedure without seriously affecting the generality of the analysis.

2. Theory of dynamic fatigue for indentation flaws

2.1. Basic fracture mechanics formulation

In this subsection we summarize the basic features of the fracture mechanics formulation [5, 6]. A schematic diagram of the model indentation flaw system is given in Fig. 1. It is assumed that the specimen surface is free of all stresses prior to indentation. The indentation event itself produces a well-defined crack system, whose characteristic size c depends on the peak contact load P. Subsequent application of a tensile stress σ_a causes the

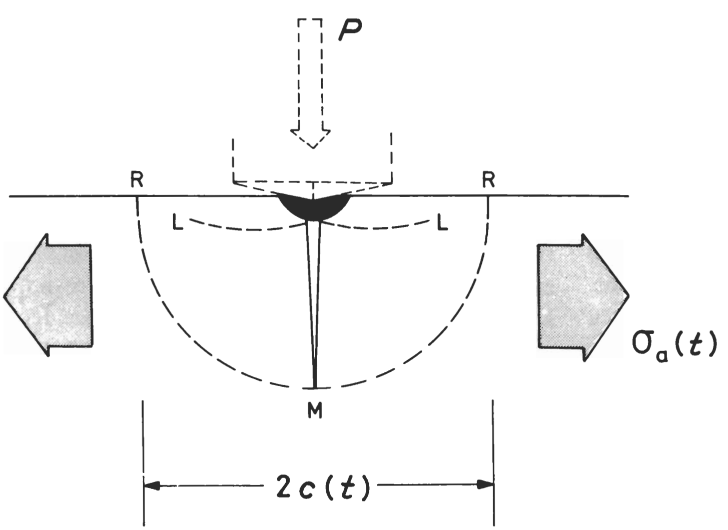

Figure 1 Schematic diagram of Vickers-produced radial/median (RM) crack system, formed at peak indentation load P and subject to expansion in characteristic dimension c under subsequent application of tensile stress σ_a. Also shown is lateral (L) crack system and central deformation zone (shaded).

crack system to grow toward an instability configuration. The crack which controls the strength is of the "radial/median" type [7]. For the Vickers indentation geometry depicted in Fig. 1 there are two such cracks, orthogonal to each other and semicircular about the origin of contact; both may expand under the action of the applied stress (depending on whether this stress is uniaxial or biaxial), but we focus our attention on the one ultimately responsible for failure. A second system of cracks, of the "lateral" type [11, 12], remains passive during strength testing. All of the cracks originate from the central deformation zone, which is also the source of the residual driving force. The stress intensity factor for the dominant radial/median crack may then be written

$$K = \frac{\chi_r P}{c^{3/2}} + \sigma_a(\pi\Omega c)^{1/2} \qquad (c \geqslant c_0), \quad (1)$$

i.e. as the sum of residual-contact and applied-stress components: χ_r is a parameter of the elastic–plastic indentation field, determined for any given material by the ratio of hardness to Young's modulus [7]; Ω is a crack geometry parameter, equal to $4/\pi^2$ for an ideal centrally-loaded penny crack [13] but here modified by the presence of crack neighbours, deformation zone and specimen free surface. The crack length c_0 pertains to the immediate post-indentation configuration, at which point the equilibrium condition $K = K_c$, where K_c is the toughness, is satisifed [7]. Putting $\sigma_a = 0$ in Equation 1 then gives $c_0 = (\chi_r P/K_c)^{2/3}$ as an initial condition for the ensuing strength test. In reality, it is not easy to avoid exposing the indented surface to a reactive environment (especially moisture, in the case of many ceramics), so the crack may extend subcritically to some non-equilibrium size c_0' prior to application of the tensile stress [3–6]. Also, the parameters χ_r and Ω may be subject to departures from constancy during the various stages of crack evolution [5, 14], suggesting that care needs to be exercised in specifying the terms in Equation 1.

Before considering fatigue effects in any detail it is useful to treat the special case where the strength test is carried out in an inert environment. The resulting "inert strength" then serves as a convenient baseline for data reduction. Inserting the equilibrium requirement $K = K_c$ into Equation 1 and solving for σ_a gives

$$\sigma_a = \left[\frac{K_c}{(\pi\Omega c)^{1/2}}\right]\left[1 - \frac{\chi_r P}{K_c c^{3/2}}\right]. \qquad (2)$$

The function $\sigma_a(c)$ passes through a maximum, σ_m, at

$$\sigma_m = \frac{3K_c}{4(\pi\Omega c_m)^{1/2}} \qquad (3a)$$

with

$$c_m = \left(\frac{4\chi_r P}{K_c}\right)^{2/3}. \qquad (3b)$$

In the crack-size range $c_0 \leqslant c < c_m$ the equilibrium remains stable, so the crack undergoes a stage of precursor growth as the stress is raised; at $c = c_m$ the configuration becomes unstable and failure occurs, thereby defining the inert, residual-stress-sensitive strength $\sigma_a = \sigma_m = \sigma_i$. The idealized Griffith crack follows as the limiting case $\chi_r = 0$ in Equation 1, whence failure occurs spontaneously at the initial crack size $c = c_0'$, defining the inert, residual-stress-free strength $\sigma_a = \sigma_i^0 = K_c/(\pi\Omega c_0')^{1/2}$. It is noted that c_0' does not appear in Equation 3, i.e. the inert strength does not depend on the initial conditions, provided $c_0' < c_m$.

Turning now to dynamic fatigue, we allow that crack growth can proceed along a subcritical path $K < K_c$ according to some rate-dependent condition. This condition is generally expressed in terms of a crack velocity function, $v(K)$, most simply in power-law form

$$v = v_0\left(\frac{K}{K_c}\right)^n \qquad (K < K_c), \qquad (4)$$

where v_0 and n are constants to be determined empirically for any given material/environment system. At $K > K_c$ the crack expands relatively rapidly, limited only by the inertia of the system. Generally, the $v(K)$ curve in the subcritical domain has more than one branch, corresponding to different mechanisms of rate control [1]. Three regions are commonly distinguished: (i) Region I, at low K, with velocity controlled by rate of reaction between environmental species and crack-tip bonds (with the possibility of a zero-velocity threshold in K, corresponding to a fatigue limit in the strength); (ii) Region II, at intermediate K, a transport-controlled region where the velocity curve tends to a plateau; (iii) Region III, at high K, a steeply rising section of the curve, independent of the environment. Of these three regions it is Region I which is usually the most important, since the crack kinetics are determined predominantly by the stages of slowest growth. Thus, writing $\sigma_a = \dot{\sigma}_a t$, with the stress rate $\dot{\sigma}_a$ constant in dynamic fatigue,

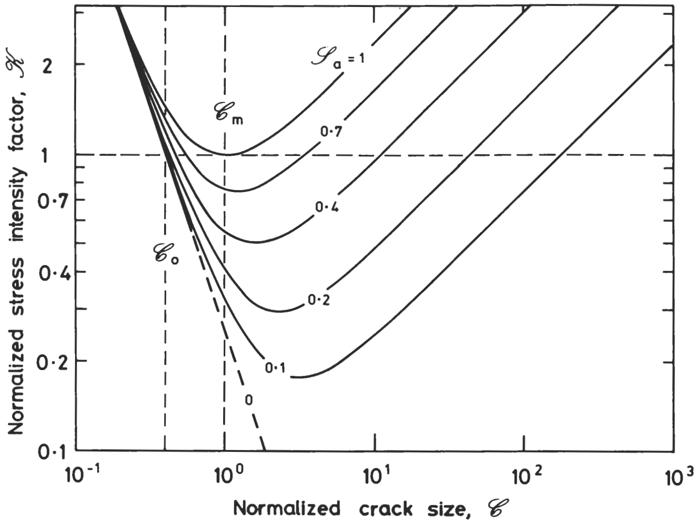

Figure 2 Normalized plot showing stress intensity factor for indentation flaw as a function of characteristic dimension, at several levels of applied stress.

fatigue study of indentation flaws in glass [6] it was shown that the time to failure was insensitive to whether c_0 or c_0' was taken as the starting dimension (with c_0' typically ≈ 50% in excess of c_0). The explanation for this behaviour lies in the fact that the function $\mathscr{K}(\mathscr{C})$ in Fig. 2 has a minimum between $\mathscr{C}_0$ and $\mathscr{C}_f$; the crack velocity is therefore lowest in the intermediate region, which accordingly controls the kinetics. We thus appear to be justified in taking $\mathscr{T} = 0$, $\mathscr{C} = \mathscr{C}_0 = 0.397$ as an invariant initial condition, rather than having to specify $\mathscr{C}_0'$ as a further adjustable. A proviso for this step to remain a good approximation is that the condition $\mathscr{C}_0' < 1$ be satisfied, corresponding to $c_0' < c_m$ in absolute terms; we recall from Section 2.1 that this is the same condition that needs to be satisfied in order that the inert strength be independent of starting flaw size.

Fig. 3 shows the results of numerical solutions of Equation 5, using a Runge–Kutta procedure,* plotted logarithmically in the standard form $\mathscr{S}(\dot{\mathscr{S}}_a)$ for selected values of n appropriate to a single-region crack velocity function. It is noted that each curve becomes closely linear in the fatigue region $\mathscr{S} < \mathscr{S}_i$ (the more so at higher n) in which case we may write, in direct analogy to Equation 6,

$$\mathscr{S} = (\Lambda' \dot{\mathscr{S}}_a)^{1/(n'+1)} \quad (\mathscr{S} < \mathscr{S}_i), \qquad (10)$$

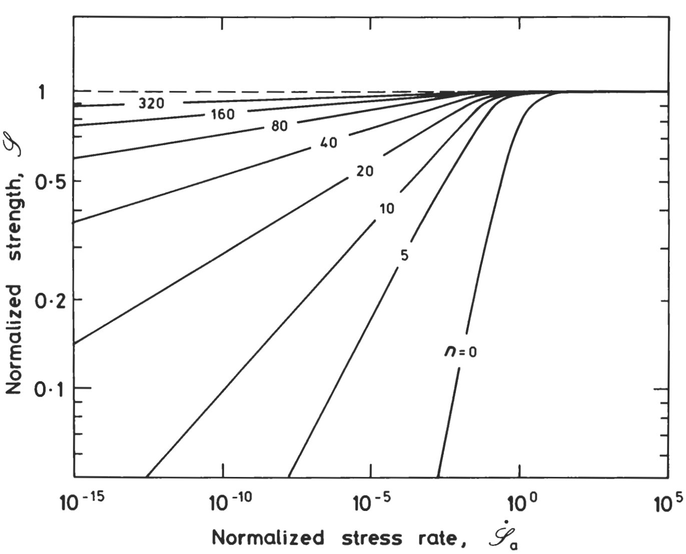

Figure 3 Normalized dynamic fatigue curves for as-indented specimens, computed for selected values of n.

*See any standard text on numerical methods.