CONTACT MECHANICS AND WEAR OF RAIL/WHEEL SYSTEMS
University of Waterloo Press, 1982

Vancouver
British Columbia
July 6-9, 1982

CONTACT FRACTURE IN BRITTLE MATERIALS

B.R. Lawn and S.M. Wiederhorn

Fracture and Deformation Division
National Bureau of Standards
Washington, D.C., U.S.A.

ABSTRACT: The nature of contact-induced surface damage in brittle materials, and the fracture mechanics principles used to describe this damage, are surveyed. The importance of understanding the elastic and plastic deformation processes which precede fracture is emphasized. Strength and erosive wear properties are intimately connected to the contact damage mechanics.

RÉSUMÉ: La nature de la détérioration par contact de la surface des matériaux fragiles et les principes de mécanique des fractures utilisés pour décrire cette détérioration sone rappelés. On souligne l'importance de la compréhension des processus de déformation élastique et plastique qui précèdent la fracture. La force et les propriétes d'érosion sont intement lies aux mécanismes de dommage par contact.

INTRODUCTION

The ubiquitous surface damage that characterises highly brittle materials, notably glasses and ceramics (i.e., solids with covalent/ionic bonding), is due to local stress concentrations that occur whenever contact is made with a small, hard object. Microfracture centers that seriously degrade the strength are often introduced by the processes used to finish the surfaces (e.g., machining), or by particle impingement incurred in subsequent handling and storage. Unless extreme precautions are taken to avoid all spurious contact events (e.g., as is done with the coating of freshly drawn optical fibres in dust-free atmospheres) such degradation is generally inevitable. Contact damage also holds the key to the erosive wear and abrasion properties of brittle materials. A proper understanding of the underlying mechanisms of deformation and fracture, using the controlled methods of "indentation fracture mechanics", has accordingly become a major research goal in the area of brittle design.

In the present paper the current state of this understanding is summarized. Our goal is not a comprehensive survey of the field: rather, we seek to draw attention to certain broad features of the brittle indentation problem that might be considered to bear, however indirectly, on the theme of this meeting. Reference is made to several review articles [1-4] for those who wish to pursue the subject in greater detail.

The outline of our presentation is as follows. First we define what we mean by a brittle solid. We then argue that indentation events can be classified into two main types, "blunt" or "sharp", according to whether the material response to fracture is essentially elastic or plastic. In normal loading the cracks are shown to have well-defined, penny-like geometries; superposition of a tangential loading component modifies these geometries significantly. Fracture mechanics relations are given for some of the more important of the crack geometries. Finally, the role of indentation fracture descriptions in formulating theories of strength and wear is discussed.

BRITTLE MATERIALS IN RELATION TO THE CONTACT PROBLEM

Ideally brittle materials are, by definition, essentially characterized by a completely elastic response up to the point of fracture. In certain instances stresses and strains close to the theoretical strength of the molecular structure can be sustained without detectable signs of permanent deformation. Coated silica glass fibers, for example, show complete recovery after undergoing tensile strains of up to 15%. The materials which fall most readily into this category are those with large components of covalent bonding, for which there exists a strong intrinsic resistance to shear-activated deformation processes [5].

However, even the most brittle of materials can, if subjected to sufficiently large constraining hydrostatic compressions to inhibit the onset of fracture, be deformed irreversibly [6]. (This statement is, of course, a self-evident truth to those concerned with the geomechanical behaviour of rocks.) Hardness indentations provide us with the simplest means of demonstrating this phenomenon; the stress field immediately beneath the contact area is intensely compressive, with a substantial component of superposed shear [7]. Thus residual impressions can be made on the surface of any material, including diamond, with a suitably penetrative indenter.

Whereas in metals the nature of the deformation processes which operate within the contact zone is reasonably well understood, in some of the more brittle materials the analogous processes remain obscure. It is clear from the magnitude of the contact pressures that the harder, covalent structures are being stressed to their theoretical limit. At this level the classical descriptions of slip by dislocation motion no longer strictly apply; instead it becomes more useful to consider the deformation modes in terms of an extended, cooperative breakdown of the structure. This is not to say that structures which undergo this kind of deformation are incapable of being "dislocated" by shear processes. Indeed, structural dislocations have been clearly identified in transmission electron microscopy observations by Hockey at indentation sites in a number of hard crystalline materials [8-10]. However, the configurations observed do not always correspond to normal crystallographic slip planes or directions. In fact, the recent identification of analogous shear processes in soda-lime glass [11] would appear to indicate that crystallographic considerations are no longer of primary importance in the constrained deformation of this class of solid.

A characteristic feature of the contact deformation zone in highly brittle materials is its strong confinement to the region immediately below the surface impression. There is no mechanism for relaxation of the "plastic" strains as there is in most metals, where extensive, long-range slip or twinning can usually occur without obstruction. Instead, these strains have to be accommodated elastically by the surrounding matrix. Consequently, high-intensity residual stress fields can develop, and these fields can exert a strong influence on subsequent mechanical response of the material.

A second characteristic feature of the contact process in brittle materials is the great ease with which microcracks initiate and propagate. In a covalent material like silicon, for instance, it is almost impossible to produce crack-free impressions, with even the most delicate of routine hardness testing machines. In this context it may be noted that a tensile stress component, however small in comparison to the hydrostatic compression within the deformation zone, is generally unavoidable in the matrix contact field [1].

BLUNT VERSUS SHARP CONTACT

The nature of the stress field in an indentation experiment depends strongly on the geometry of the contacting surfaces, as well as on the mechanical properties of the materials involved. We shall be working on the premise here that the indenter material is sufficiently "hard" relative to that of the test piece to be effectively rigid. It is then convenient to distinguish two extreme types of indentation field [1]: "blunt", in which the contact pressure increases monotonically with load such that the deformation prior to fracture is completely elastic; "sharp", in which the contact pressure is in excess of that required to produce irreversible deformation at all stages of loading. The intensity of the stress field in the former case is controlled by the elastic moduli, in the latter by a combination of the elastic moduli and the hardness.

Blunt Contact

The classical example of the blunt contact is the Hertzian stress problem, discussed at some length by others in this volume. Experimentally, the Hertzian stress field is most simply generated by pressing a sphere onto a semi-infinite solid. Solutions for the field are obtained from the basic equations of linear elasticity. Essentially, the normal stresses are all highly compressive in a drop-shaped zone immediately below the indenter, but become moderately tensile at and outside the contact circle. These tensile stresses are extremely inhomogeneous in this near-contact region, falling off dramatically along subsurface stress trajectories [1,12]. Remote from the contact zone the stress field tends asymptotically to the corresponding Boussinesq field for concentrated point loading [1,13].

At a critical load in the Hertzian contact a well-defined, cone-shaped crack "pops in" from the specimen surface. Figure 1 shows such a Hertzian fracture in glass.

The stresses at the surface trace prior to pop-in are generally well below the theoretical limiting strength of the material structure, indicating that initiation must occur from pre-present flaws. The crack first runs from the critical surface flaw into a shallow ring just outside the contact circle, then propagates downward into its characteristic cone geometry until sufficiently remote from the loading center, at which point it becomes highly stable.

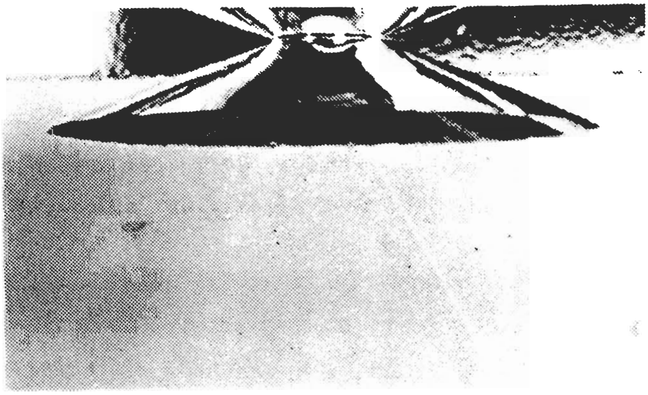


Figure 1 - Cone Crack in Glass; Base Diameter 30 mm, Indentation Load 40 MN. After [15].

The mechanics of formation of Hertzian cone cracks is complicated by the extreme inhomogeneity of the near-contact stress field through which the growth occurs. To ignore this inhomogeneity and assume that instability ensues when the surface stresses reach the tensile strength of the material is to overlook the essence of the general contact fracture phenomenon. In accordance with modern-day fracture mechanics procedure it is necessary to compute a "stress intensity factor", representing the driving force for the fracture, as an integral of actual stresses (weighted with an appropriate Green's function) over the prospective crack path. The first such analysis was carried out by Frank and Lawn [12], who showed that the critical load for cone-crack pop-in, under equilibrium conditions of fracture, is given by

$$P_c = A_1 K_c^2 r/E \ , \tag{1}$$

where r is the sphere radius, K_c is the critical stress intensity factor for crack extension (i.e., the material "toughness", characterizing the intrinsic resistance to fracture), E is Young's modulus and A_1 is a dimensionless constant. This result provided the first analytical derivation of the long-standing empirical "law" of Auerbach, 1891, $P_c \propto r$ [14]. Considerable interest has been shown in this law because the Hertzian equations for the surface stresses give $\sigma \propto P/r^2$ so that, in combination with equation (1), we obtain $\sigma_c \propto 1/r$; i.e., the simplistic concept of a critical stress criterion for fracture is clearly in violation. Note that this last expression implies the suppression of crack initiation at small sphere radii - the possibility of inducing precursor plasticity thus becomes stronger for "sharper" indenters.

The size to which the cone crack grows, once initiated, is determined by the far-field conditions. Roesler [15], using a dimensional analysis argument, showed that the crack size increases with load according to $c \propto P^{2/3}$. A more detailed, fracture mechanics treatment [16] gives, at equilibrium,

$$P/c^{3/2} = A_2 K_c \, , \qquad (P > P_c) \, , \tag{2}$$

where A_2 is another dimensionless constant.

It is interesting to note that neither of equations (1) and (2) is sentitive to the size of the critical flaw from which the cone crack initiates. This insensitivity is another characteristic of the general contact problem, although it is implicit in the derivation of the above expressions that there is always a sufficiently high density of surface flaws present to guarantee the initiation condition. It may also be noted that the formulations contain the toughness K_c, the definition of which strictly implies a configuration of mechanical equilibrium (unstable in relation to equation (1), stable in equation (2)). In practice, brittle materials are susceptible to chemically-enhanced rate-dependent crack growth, particularly in the presence of atmospheric moisture, in which case equation (1) tends to overestimate the critical load and equation (2) to underestimate the crack size.

The mechanics of fracture are modified somewhat by the superposition of a tangential force component onto the normal loading configuration. Stress solutions for the case of complete slippage at a sliding contact interface have been described by Hamilton and Goodman [17]. The main effect of the tangential force is to enhance the tensile stresses at the trailing edge of the advancing sphere, thereby destroying the axial symmetry of the field. Consequently, the critical load for fracture is reduced, dramatically at the higher coefficients of friction, and the cracks form only partial (highly distorted) cones [18]. The modified field for a coefficient of friction 0.1 is shown in Figure 2, together with a micrograph for the corresponding conditions in a sliding friction test on a glass surface [19]. Formulations analogous to equations (1) and (2) for the sliding sphere have been attempted by several workers, but a strong sensitivity to starting assumptions in the analysis has led to considerable divergence in the various predictions, particularly in the critical load.

Sharp Contact

We alluded in the previous subsection to the increasing prospect of precursor inelastic deformation with diminishing indenter radius. In the limit of zero radius, e.g., as with an ideally sharp Vickers or Knoop pyramidal indenter, such deformation becomes unavoidable; a finite load cannot be supported by a point contact without exceeding the elastic limit. The stress field beneath the indenter is considerably more complex than in its Hertzian elastic counterpart, and simplistic elastic/plastic models have to be devised to provide a necessary framework for fracture mechanics analysis. The simplest and most widely adopted of the models is that of the expanding internal cavity, in which a pressurized spherical volume (hardness impression) induces plasticity in an immediate annular surround volume (deformation zone), the whole being constrained in an infinite elastic matrix [20,21]. Despite clear shortcomings, e.g., failure to allow for stress relaxation at the specimen free surface and failure to match the Boussinesq point-force solution in the far field, models of this type are useful for their amenability to closed-form solution.

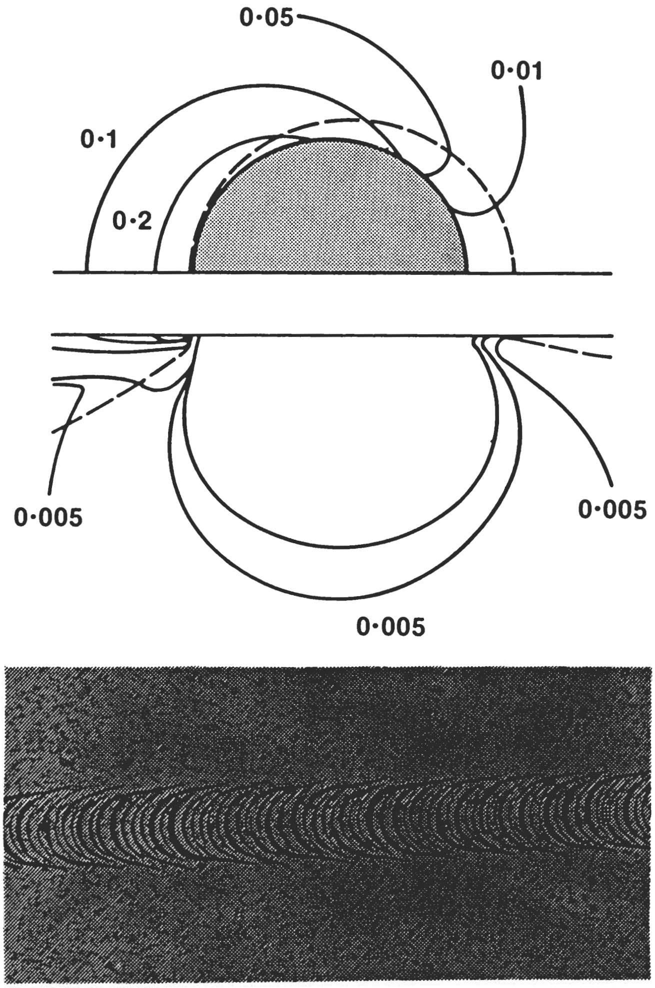


Figure 2 - Sliding Contact Fracture Beneath Sphere, Friction Coefficient 0.1. Schematic half-surface and side views show contours of maximum tension (solid curves) and principal stress trajectories (dashed curves) about contact circle (shaded). After [18]. Micrograph shows partial cone track on glass, width 25 μm, produced by sphere of radius 1.5 mm at load 10 N. After [19].

The fracture pattern produced in sharp indentation has some geometrical variants, but two basic crack types are generally distinguishable; median/radial, and lateral, cracks [13]. Figure 3 is an illustrative example, again in glass. It is found that the initiation of such cracks is accomplished with relative ease in highly brittle materials, even on surfaces of the greatest perfection, suggesting that the precursor deformation processes are capable of producing their own fracture nuclei [11]. Each median crack forms on a symmetry plane containing the contact axis and an impression diagonal, and leaves a characteristic radial trace on the indented surface; as seen in Figure 3, more than one crack of this type can be formed in a given contact, depending on the indenter geometry. Lateral cracks form in a saucer-like manner, starting from near the base of the deformation zone and spreading outward closely parallel to the specimen surface; in severe cases the laterals can extend to the surface, causing chipping. As with the Hertzian cones these cracks, once developed, are highly stable.

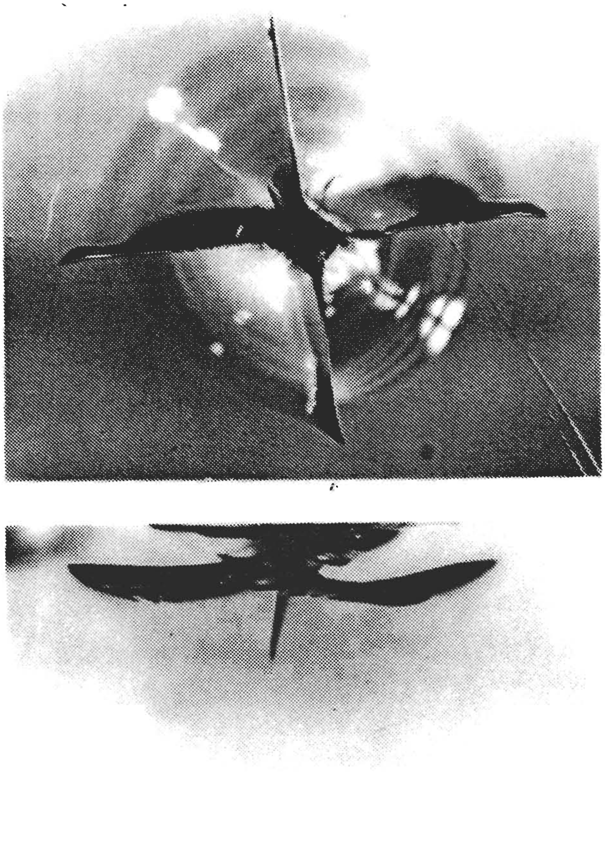


Figure 3 - Median/Radial/Lateral Fracture System for Vickers Indentation on Glass; Radial Crack Diameter 1.3 mm, load 100 N. Courtesy of B.J. Hockey.

The conditions for the initiation of indentation cracks beneath sharp indenters have only recently been considered. Lawn and Evans [22] used a fracture mechanics procedure somewhat analogous to that in the earlier cone crack analysis [12], but with the intensity of the stress field now determined by a load-invariant hardness H. In their scheme fracture occurs when the spatial extent of the field, as determined by the contact dimension, is sufficiently large to make available flaws within the deformation zone unstable. Accordingly, the critical load is found to be

$$P_c = A_3(E/H)(K_c/H)^3K_c\ , \tag{3}$$

where A_3 is a dimensionless term involving the characteristic indenter half-angle. Whereas the analysis is acknowledged to be deficient in its ability to specify the absolute value of A_3, Equation (3) serves well as a basis for ranking materials. For example, comparison of a moderately hard steel with a typical silicate glass leads to a predicted ratio of $\approx 10^3 - 10^4$ in P_c values, thus explaining the brittleness of the latter [7].

In considering the mechanics of well-developed median and lateral cracks we can take note of one geometrical feature common to all far-field indentation fracture configurations; their ultimate fronts tend to be circular, so they have the essential character of center-loaded penny cracks [16]. For the median cracks, which approximate to half-pennies centered on the point of contact, detailed analysis provides a relation between load P and crack radius c, [23]

$$P/c^{3/2} = A_4(H/E)^{1/2}K_c\ , \qquad (P > P_c) \tag{4}$$

in direct analogy to equation (2), with A_4 again incorporating the indenter half-angle. However, if the final configuration of the median crack bears a strong underlying resemblance to that of its cone crack counterpart, its route to this configuration during the indentation cycle is of a totally different nature. For, in addition to the usual crack driving force associated with the reversible (elastic) component of the stress field, there is also a contribution from the irreversible (plastic) component, and it is the second of these components which dominates in the median crack evolution. Thus, much of the crack evolution occurs as the indenter is being unloaded, particularly along the radial traces in the near-surface region [23,24]. Indeed, in materials susceptible to moisture-enhanced crack growth, the radial crack traces can be observed to continue extending long after the indenter has been removed.

Derivation of a similar expression for lateral cracks is even less straightforward, owing to the added complication of the nearby specimen free surface. Whereas for the more penetrative median and cone cracks the free surface is unlikely to have a strong influence on the strain energy distributions which govern crack extension, the same is certainly not true of the highly compliant material portions immediately above the lateral plane. Thus, although the lateral system retains the penny-like character, it is more realistic to regard it in terms of a thin, circular disc of material built in at its circumference to a rigid matrix rather than in terms of the usual embedded configuration. Using the theory of elastic plates to determine the energetics of this system a result analogous to equation (4) can be derived [25], except that now the relation between crack size and load is of the form $c \propto P^{5/8}$ and the dependence on material parameters is somewhat more complex. Again, the bulk of the crack growth takes place during indenter withdrawal, re-emphasising the crucial role of residual stresses when inelastic deformation accompanies fracture.

When a sharp indenter is translated across the surface of a brittle material it leaves a scratch track. Again, the introduction of a frictional force, this time determined by a "ploughing" process, enhances the fracture development [26]. In environment-sensitive materials rate variables, such as scratch velocity, become a factor. In accordance with the arguments leading to equation (3) it is found that "light" loads tend to produce smooth scratches, "heavy" loads fragmented scratches. In the former case the action is one of "polishing", in the latter of "abrasion" [1]. Figure 4 shows an example of a scratch made just above the threshold load in glass. (Note that a cursory examination of the surface in this case would create the false impression of a purely plastic contact event.) Comparison of Figure 4 with Figure 3 shows that the sliding motion tends to suppress one set of median cracks, thus producing an essentially linear flaw. At excessively high loads extensive chipping occurs along the length of the scratch as the laterals turn upward to intersect the surface. This can lead to physical removal of the deformation zone, and thereby of the source of the residual driving force on the sub-survace median crack.

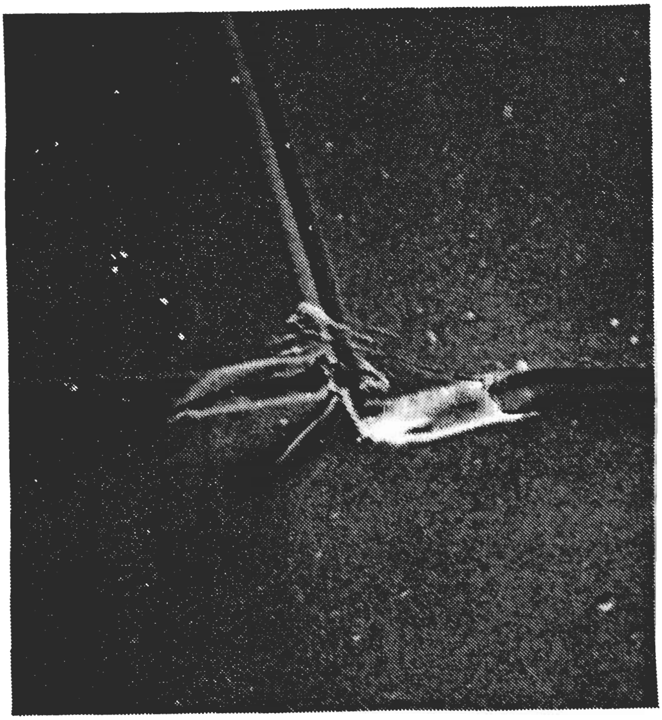


Figure 4 - Scratch on Glass, ≈ 10 μm wide, made by sliding Vickers Indenter at load 1 N. After [26].

PRACTICAL APPLICATIONS

Extensive experimental confirmation of the preceding theoretical formulations has provided us with a sound foundation for modelling various practical processes in brittle glasses and ceramics. We shall focus our attention on two of the more important of these processes, strength degradation and erosive wear. In the first case it is the penetrative cone or median crack which is the governing element; in the second case the lateral system dominates.

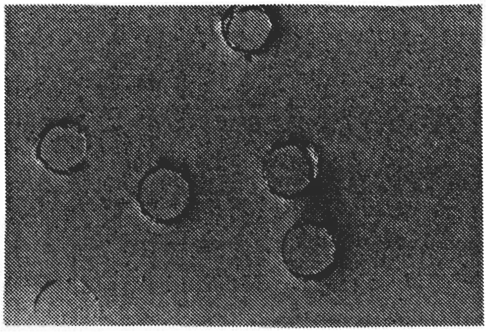
Strength Degradation

The strength of a brittle material is controlled by the size of the largest flaw in the surface or bulk. In many cases this dominant flaw results from a well-characterized contact event, e.g., from machining during fabrication or particle impact in service. Failure then becomes a function of the contact variables, which can often be accurately specified. This accuracy can be optimized by deliberately introducing controlled indentation cracks into prospective test pieces. Apart from circumventing many uncertainties associated with the usual statistical approach to the strength of materials with unclassified flaws, this course opens up many avenues to systematic materials evaluation [4]. Moreover, by observing how "macroscopic" contact-induced cracks respond under an applied tensile stress we can gain valuable physical insight into the nature of "microscopic" natural flaws which are generally undetectable.

As an example of the type of information that can be obtained from strength degradation studies we present results from a ballistic impact study on glass surfaces using steel spheres, Figure 5 [27]. The micrograph shows the damage incurred at one specified impact velocity, and the graph shows how the remaining strength varies systematically with this velocity. The solid curve through the data points is a prediction from the conventional strength formula for spontaneous failure from flaws of characteristic size c_0,

$$S = K_c/Yc_0^{1/2} \tag{5}$$

where Y is a geometrical constant close to unity; c_0 is evaluated in accordance with equations (1) and (2) for blunt indenters, using the Hertzian contact theory to eliminate load as an unknown in favour of velocity [27]. We note that no degradation occurs in Figure 5 below the threshold velocity for cone crack pop in, at which point the strength drops abruptly; beyond the threshold the strength falls off steadily with increasing velocity.



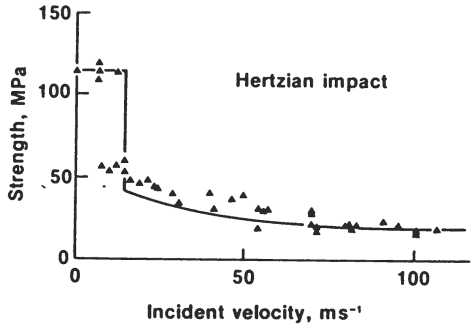


Figure 5 - Strength Degradation of Glass Impacted by Spheres. Micrograph shows surface damage after impact with 35-mesh glass beads; diameter of cone crack surface trace 400 μm, incident velocity 100 ms^{-1}. Graph shows remaining strength as function of impact velocity with steel spheres, diameter 0.8 mm. After [27].

The strength behaviour of materials containing median flaws induced by sharp contact is more complicated, owing to the active role of residual stresses from the plastic enclave in post-contact crack growth [28]. As a result of a strong stabilizing influence of the residual component in the net driving force for fracture, the medians do not propagate spontaneously to failure in the manner of cone cracks, but rather undergo a precursor stage of growth from c_0 to a critical size c_m before attaining an instability configuration. Some data taken from direct observations of radial crack growth during application of a tensile stress to Vickers indented glass test pieces is plotted in Figure 6. The critical applied stress σ_m defines a new strength level [28]

$$S = K_c/2Yc_m^{1/2} , \tag{6}$$

which is always less than the evaluation from equation (5). Hence the presence of residual stresses has a clearly detrimental effect on the structural integrity of materials. It can be shown from the fracture mechanics that $c_m \propto (P/K_c)^{2/3}$, independent of the initial flaw size c_0, so that control of strength can be exercised via the indentation load P.

It is this feature which establishes equation (6) as an attractive base formula for the accurate determination of material fracture parameters from strength data [4].

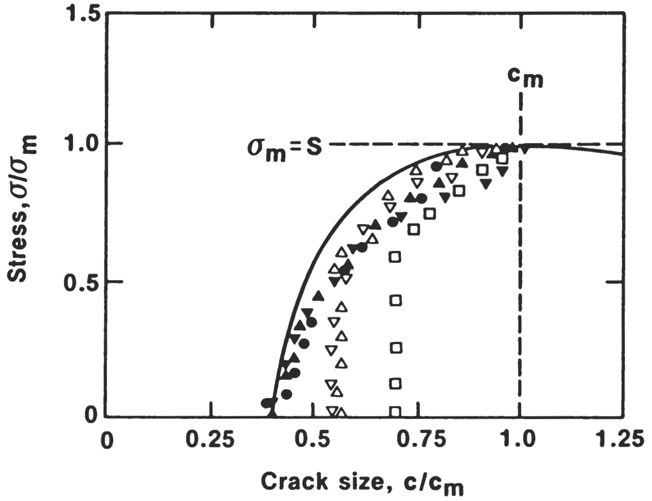


Figure 6 - Stable Crack Extension in Vickers-Indented Glass Prior to Failure in Tensile Loading. Stress and crack size normalized to coordinate values at maximum in curve. Closed symbols denote specimens confined to inert environment prior to failure test to suppress post-indentation crack creep, open symbols denote corrosive (moist) environment. Note failure conditions insensitive to initial crack size. Courtesy of D.B. Marshall.

One area of particularly intense recent interest in relation to the issue of materials evaluation is that of "fatigue", which in the ceramics testing fraternity is taken to mean strength loss due to environmentally-assisted crack growth during a static or monotonically increasing applied tensile stress. The introduction of indentation flaws into fatigue test pieces allows for determination of appropriate rate parameters for the underlying crack growth process, with unprecedented experimental simplicity and specimen economy [29,30]. More significantly in the present context, however, it highlights the dramatic increase in susceptibility to fatigue effects that flaws exhibit when residual stresses are present. For instance, by annealing out such stresses from Vickers-indented glass test pieces the lifetime at a prescribed applied load is extended by some three orders of magnitude [31]. There are important implications here in establishing design criteria for brittle materials, especially in view of a growing conviction that many "natural" flaws may be more closely simulated by indentation cracks in their "as-produced" rather than "annealed" state.

At the time of writing, studies of strength degradation for indenters subject to translational motion have only just begun. Early indications are that the linear flaws produced in this type of contact are likely to be even more strongly influenced by residual stress effects than are the simple point-contact flaws described above.

Erosive Wear

The fracture mechanics relations derived earlier can also be used to develop models for erosive wear, where one seeks to inhibit cracking, and for machining, where one seeks to promote cracking. Sharp indenters are most effective in such material removal processes because of the relative ease with which lateral cracks form intersections with the free surface. Accordingly, most treatments of erosive wear and machining damage have concentrated on this contact mode. We shall consider just one practical example, that of erosion by normally incident sharp particles, to illustrate the modelling procedure.

The basic starting equation for the formulation defines the approximate amount of material removed in a single impact event,

$$\Delta V = \pi c^2 d \ , \tag{7}$$

where c is the radius and d the depth of the subsurface lateral crack. The crack radius is estimated from the lateral crack analogue of equation (4), and the depth from the penetration of the deformation zone. An appropriate impulse relation is once again required to eliminate contact load in favour of incident velocity. Summation over all impacts, assuming zero interaction between neighbours, leads to a functional relation (usually in power-law form) for the total volume removal rate in terms of the particle kinetic energy and material parameters. Unfortunately, the final expression obtained is extremely sensitive to minor details in the analysis (note equation (7) involves the cube of a linear dimension, whereas the strength formulae of equations (5) and (6) involve only the square root), and erosive data are notorious for their scatter, the result of which is an accumulation of slightly different, experimentally indistinguishable theories [32,33]. Thus the complexity of the erosion process admits only a partial theoretical account of experimental behaviour.

ACKNOWLEDGEMENT

This project was supported by the Office of Naval Research, Metallurgy and Ceramics Program.

REFERENCES

[1] LAWN, B.R. and WILSHAW, T.R., "Indentation Fracture: Principles and Applications", *J. Mater. Sci.*, Vol. 10, 1975, p. 1049.

[2] LAWN, B.R. and MARSHALL, D.B., "Indentation Fracture and Strength Degradation of Ceramics", *Fracture Mechanics of Ceramics*, Edited by R.C. Bradt, D.P.H. Hasselman and F.F. Lange, Plenum Press, New York, Vol. 3, 1978, p. 205.

[3] LAWN, B.R. and MARSHALL, D.B., "Mechanisms of Microcontact Fracture in Brittle Solids", *Lithic Use-Wear Analysis*, Edited by B. Hayden, Academic Press, New York, 1979, p. 63.

[4] LAWN, B.R., "The Indentation Crack as a Model Surface Flaw", *Fracture Mechanics of Ceramics*, Edited by R.C. Bradt, D.P.H. Hasselman, F.F. Lange and A.G. Evans, Plenum Press, New York, 1982, in press.

[5] KELLY, A., *Strong Solids*, Clarendon, Oxford, 1966, Chapters 1 and 3.

[6] BRIDGEMAN, P.W., *The Physics of High Pressures*, G. Bell, London, 1949.

[7] LAWN, B.R. and MARSHALL, D.B., "Hardenss, Toughness and Brittleness: An Indentation Analysis", *J. Amer. Ceram. Soc.*, Vol. 62, 1979, p. 347.

[8] HOCKEY, B.J., "Plastic Deformation of Aluminum Oxide by Indentation and Abrasion", *J. Amer. Ceram. Soc.*, Vol. 54, 1971, p. 223.

[9] HOCKEY, B.J. and LAWN, B.R., "Electron Microscopy of Microcracking About Indentations in Aluminum Oxide and Silicon Carbide", *J. Mater. Sci.*, Vol. 10, 1975, p. 1275.

[10] LAWN, B.R., HOCKEY, B.J. and WIEDERHORN, S.M., "Atomically Sharp Cracks: An Electron Microscopy Study", *J. Mater. Sci.*, Vol. 15, 1970, p. 1207.

[11] HAGAN, J.T., "Shear Deformation Under Pyramidal Indentations in Soda-Lime Glass", *J. Mater. Sci.*, Vol. 15, 1980, p. 1417.

[12] FRANK, F.C. and LAWN, B.R., "On the Theory of Hertzian Fracture", *Proc. Roy. Soc. London*, Vol. A299, 1967, p. 291.

[13] LAWN, B.R. and SWAIN, M.V., "Microfracture Beneath Point Indentations in Brittle Solids", *J. Mater. Sci.*, Vol. 10, 1975, p. 113.

[14] AUERBACH, F., "Measurement of Hardness", *Ann. Phys. Chem.*, Vol. 43, 1891, p. 61.

[15] ROESLER, F.C., "Brittle Fractures Near Equilibrium", *Proc. Phys. Soc. London*, Vol. B69, 1956, p. 981.

[16] LAWN, B.R. and FULLER, E.R., "Equilibrium Penny-Like Cracks in Indentation Fracture", *J. Mater. Sci.*, Vol. 10, 1975, p. 2016.

[17] HAMILTON, G.M. and GOODMAN, L.E., "The Stress Field Created by a Circular Sliding Contact", *J. Appl. Mech.*, Vol. 33, 1966, p. 371.

[18] LAWN, B.R., "Partial Cone Crack Formation in a Brittle Material Loaded with a Sliding Spherical Indenter", *Proc. Roy. Soc. London*, Vol. A299, 1967, p. 307.

[19] CRIMES, G.M., *Ph. D. Thesis*, University of Sussex, England, 1973.

[20] HILL, R., *The Mathematical Theory of Plasticity*, Oxford University Press, London, 1950, Chapter 5.

[21] JOHNSON, K.L., "The Correlation of Indentation Experiments", *J. Mech. Phys. Solids*, Vol. 18, 1970, p. 115.

[22] LAWN, B.R. and EVANS, A.G., "A Model for Crack Initiation in Elastic/Plastic Indentation Fields", *J. Mater. Sci.*, Vol. 12, 1977, p. 2195.

[23] LAWN, B.R., EVANS, A.G. and MARSHALL, D.B., "Elastic/Plastic Indentation Damage in Ceramics: The Median/Radial Crack System", *J. Amer. Ceram. Soc.*, Vol. 63, 1980, p. 574.

[24] MARSHALL, D.B. and LAWN, B.R., "Residual Stress Effects in Sharp Contact Cracking: I. Indentation Fracture Mechanics", *J. Mater. Sci.*, Vol. 14, 1979, p. 2001.

[25] MARSHALL, D.B., LAWN, B.R. and EVANS, A.G., "Elastic/Plastic Indentation Damage in Ceramics: The Lateral Crack System", *J. Amer. Ceram. Soc.*, in press.

[26] SWAIN, M.V., "Microfracture about Scratches in Brittle Solids", *Proc. Roy. Soc. London*, Vol. A366, 1979, p. 575.

[27] WIEDERHORN, S.M. and LAWN, B.R., "Strength Degradation of Glass Resulting from Impact with Spheres", *J. Amer. Ceram. Soc.*, Vol. 60, 1977, p. 451.

[28] MARSHALL, D.B., LAWN, B.R. and CHANTIKUL, P., "Residual Stress Effects in Sharp Contact Cracking: II. Strength Degradation", *J. Mater. Sci.*, Vol. 14, 1979, p. 2225.

[29] LAWN, B.R., MARSHALL, D.B., ANSTIS, G.R. and DABBS, T.P., "Fatigue Analysis of Brittle Materials Using Indentation Flaws: I. General Theory", *J. Mater. Sci.*, Vol. 16, 1981, p. 2846.

[30] COOK, R.F., LAWN, B.R. and ANSTIS, G.R., "Fatigue Analysis of Brittle Materials Using Indentation Flaws: II. Case Study on a Glass Ceramic", *J. Mater. Sci.*, Vol. 17, 1982, p. 1108.

[31] CHANTIKUL, P., LAWN, B.R. and MARSHALL, D.B., "Micromechanics of Flaw Growth in Static Fatigue: Influence of Residual Contact Stresses", *J. Amer. Ceram. Soc.*, Vol. 64, 1981, p. 322.

[32] RUFF, A.W. and WIEDERHORN, S.M., "Erosion by Solid Particle Impact", *Treatise on Materials Science and Technology: Erosion,* edited by C.M. Preece, Academic Press, New York, Vol. 16, 1979, p. 69.

[33] WIEDERHORN, S.M. and HOCKEY, B.J., "Effect of Material Parameters on the Erosion Resistance of Brittle Materials", *J. Mater. Sci.,* in press.