

Theory of Fatigue for Brittle Flaws Originating from Residual Stress Concentrations

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A theory is formulated for the general fatigue response of brittle flaws which experience residual stress concentrations. The indentation crack is taken as a model flaw system for the purpose of setting up the basic fracture mechanics equations, but the essential results are expected to have a wider range of applicability in the strength characterization of ceramics. A starting fatigue differential equation is first set up by combining an appropriate stress intensity factor for point- or line-contact flaws with a power-law crack velocity function. Analytical solutions are then obtained for the case of static fatigue. The resulting relation between lifetime and failure stress is shown to have exactly the same power-law form as the conventional solution for Griffith (residual-stress-free) flaws. This "equivalence" is used as a basis for extending the results to dynamic fatigue. A comparison of these analytical solutions with numerical counterparts defines the limits of accuracy of the theoretical procedure. However, while the form of the lifetime relation remains invariant, the values of the exponent and coefficient differ significantly for flaws with and without residual stress. Accordingly, the application of conventional fatigue theory to evaluate crack velocity parameters, without due regard for the nature of the critical flaw, can lead to serious errors. Explicit conversion formulas are given for transforming "apparent" velocity parameters for indentation flaws directly into "true" parameters. The implications of these results concerning the use of the indentation method for materials evaluation are discussed.

I. Introduction

IT IS well recognized that the failure of brittle materials is governed by the micromechanics of crack growth from small flaws, and that chemical enhancement of this crack growth can cause significant reductions in the strength with increasing time under load. Embodied in the conventional fracture mechanics approach to "fatigue" phenomena of this kind^{1,2} are three underlying assumptions: (a) The time dependence of the loading stresses, taken to act uniformly across the prospective crack plane, is specifiable; (b) the driving force on the extending crack is uniquely determined at any given characteristic length by these applied loading stresses; (c) the rate of crack extension is in turn uniquely determined by some well-defined function of the driving force for any given material/environment system. These assumptions allow one to write down a differential equation in crack length and time, the solution of which defines the stress conditions at failure. The widespread success enjoyed by the fracture mechanics formulation arises from the amenability to solutions in simple, closed form, which provide a convenient basis for lifetime predictions.

Apart from the clear-cut distinction made between loading at constant stress ("static fatigue") and constant stress rate ("dynamic fatigue"), surprisingly little attention has been devoted to the effects that potential variations in the starting equations may have in the lifetime analysis. Wiederhorn and Ritter³ examined the crack

velocity function, and concluded that any of the commonly accepted empirical forms may fit fatigue data equally well (although extrapolations beyond the data range could lead to significant discrepancies in the predictions). Of the assumptions listed above, the second has been subjected to least scrutiny, it generally being assumed, without question, that the flaws respond in the classical "Griffith" sense; that is, the flaw is driven solely by the applied loading, this force increasing monotonically with the crack size until some instability condition is met.^{4,5} It is thus implicit in the statement of the problem that any preexisting stresses which may have been responsible for generating the critical flaw in the first place^{5,6} have long since ceased to be a significant contributing factor in the net driving force on the system.

However, recent studies of controlled flaws produced by indentation in strength test pieces have demonstrated that residual crack-generation stresses can have a profound influence on the crack evolution to failure.⁷ The source of the residual field in this case is elastic-plastic mismatch at the boundary of the deformation zone which encases the sharp point and edges of the contacting body.⁸ It then becomes necessary to incorporate a residual-contact term into the fracture mechanics equation for the crack driving force. Characteristically, this contribution *decreases* monotonically with crack size.⁸ The resulting expression for the net force on the crack now takes on a considerably more complicated form. This complexity is such that the appropriate fatigue differential equation no longer appears to have simple analytic solutions. Accordingly, the first systematic investigations of residual-stress effects in fatigue, using results from dynamic⁹ and static¹⁰ loading tests on Vickers-indented soda-lime glass in water as a data base, were made by obtaining numerical solutions specific to one particular indenter/material/environment system. A subsequent analysis,¹¹ based on a reformulation of the differential equation in terms of judiciously normalized variables, allowed for generalization of the numerical procedure to include solutions for all possible systems. Most notably, this last study produced an empirical dynamic fatigue relation, for flaws satisfying a power-law crack velocity function, which was indistinguishable in form from that derived analytically for Griffith flaws. The exponents and coefficients in this relation were not, however, identical in the two cases; in particular, the values of the fatigue exponent, which for Griffith flaws is a direct measure of the corresponding exponent in the crack velocity function, differed by some 30%. A case study on a glass-ceramic¹² confirmed these and other features of the residual-stress theory, and outlined several unique advantages of the indentation method as a means for evaluating basic fatigue parameters.

One point that must be made at the outset is that indentation cracks should not be regarded simply as artificially introduced entities which bear no resemblance to strength-controlling flaws in real materials. There is growing evidence that the degrading surface damage which many ceramic components experience in finishing (e.g. machining)¹³ or in service (sharp particle impact)¹⁴ are characterized by the same residual stress effects as are indentation flaws. Indeed, the observation of strongly analogous local stress field effects about microstructural flaws in ceramics¹⁵ suggests that the presence of residual crack driving forces may be the rule rather than the exception.

Viewed against this background the solution of the indentation fatigue problem takes on a broader significance. Accordingly, the

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